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3D-xy critical properties of $YBa_2Cu_4O_8$ and magnetic-field-induced 3D to 1D crossover

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Abstract

We present reversible magnetization data of a $YBa_2Cu_4O_8$ single crystal and analyze the evidence for 3D-xy critical behavior and a magnetic-field-induced 3D to 1D crossover. Remarkable consistency with these phenomena is observed in agreement with a magnetic-field-induced finite size effect, whereupon the correlation length transverse to the applied magnetic field cannot grow beyond the limiting magnetic length scale $L_H = (\Phi_0/(aH))^{1/2}$. By applying the appropriate scaling form we obtain the zero-field critical temperature, the 3D to 1D crossover, the vortex melting line and the universal ratios of the related scaling variables. Accordingly there is no continuous phase transition in the (H, T) plane along the H_{c2} lines as predicted by the mean-field treatment.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Fluctuation effects are known to be strongly enhanced in high temperature cuprate superconductors due to their anisotropic behavior and their high zero-field transition temperature T_c [1, 2]. For $YBa_2Cu_4O_8$ and related compounds several studies point to the importance of critical fluctuations [3–6]. To circumvent the smearing of the phase transition due to inhomogeneities $YBa_2Cu_4O_8$ is an exquisite candidate due to its nearly stoichiometric structure and the availability of excellent single crystals [7, 8]. As $YBa_2Cu_4O_8$ is intrinsically doped there is no anomalous precursor diamagnetism expected, as reported for example in aluminum-doped MgB_2 [6].

In this study we present reversible magnetization data of a $YBa_2Cu_4O_8$ single crystal and analyze the evidence for 3D-xy critical behavior and a magnetic-field-induced 3D to 1D crossover. We observe remarkable consistency with these phenomena. Since near-T_c thermal fluctuations are expected to dominate [1, 9–12], Gaussian fluctuations point to a magnetic-field-induced 3D to 1D crossover [13]. Whereby the effect of fluctuations is enhanced, it appears inevitable to take thermal fluctuations into account. Indeed, invoking the scaling theory of critical phenomena we show that the data are inconsistent with the traditional mean-field interpretation. In contrast, we observe agreement with a magnetic-field-induced finite size effect, whereupon the correlation length transverse to the magnetic field H_i, applied along the i axis, cannot grow beyond the limiting magnetic length

$$L_{H_i} = (\Phi_0/(aH_i))^{1/2}, \tag{1}$$

with $a \simeq 3.12$ [14]. L_{H_i} is related to the average distance between vortex lines. Indeed, as the magnetic field increases, the density of vortex lines becomes greater, but this cannot continue indefinitely. The limit is roughly set on the proximity of vortex lines by the overlapping of their cores. This finite size effect implies that, in type II superconductors, superconductivity in a magnetic field is confined to cylinders with diameter L_{H_i} [12, 15]. Accordingly, below T_c there is the 3D to 1D crossover line

$$H_{pi}(T) = (\Phi_0/(a\xi_{j0}^{-}\xi_{k0}^{-}))(1 - T/T_c)^{4/3}, \tag{2}$$

with $i \neq j \neq k$. $\xi_{i0,j0,k0}^{-}$ denotes the critical amplitudes of the correlation lengths below T_c along the respective axes.

It circumvents the occurrence of the continuous phase transition in the (H_c, T) plane along the H_{c2} lines predicted by the mean-field treatment [16]. Indeed, the relevance of thermal fluctuations emerges already from the reversible magnetization data shown in figure 1. As a matter of fact, the typical mean-field behavior [16], whereby the magnetization scales below T_c linearly with the magnetic field, does not emerge.

2. Experiment and analysis

The $YBa_2Cu_4O_8$ single crystal investigated in this work was fabricated by a high-pressure synthesis method described in detail elsewhere [7, 8]. The volume of the nearly rectangular shaped sample is estimated to be 3.9×10^{-4} cm^3. The magnetic moment was measured by a commercial Quantum Design DC-SQUID magnetometer MPMS XL with installed RSO option, allowing us to achieve a resolution of 10^{-8} emu. For this experiment the applied magnetic field was oriented along the c axis of the crystal. For different magnitudes of the field, temperature-dependent magnetization curves were measured. The zero-field-cooled (ZFC) and field-cooled (FC) data have been compared in order to probe the reversible magnetization only. The superconducting susceptibility was finally obtained by correcting the measured data for the normal state and sample holder contributions. In figure 1 we depicted some of the measured magnetization curves m versus T for magnetic fields H_c applied along the c axis. At a first glance the data fall on rather smooth curves, revealing that the extraction of critical and crossover behavior requires a rather detailed analysis. When thermal fluctuations dominate and the coupling to the charge is negligible the magnetization per unit volume, $m = M/V$, adopts the scaling form [1, 9–11]

$$\frac{m}{TH^{1/2}} = -\frac{Q^{\pm}k_B\xi_{ab}}{\Phi_0^{3/2}\xi_c}F^{\pm}(z),$$
$$F^{\pm}(z) = z^{-1/2}\frac{dG^{\pm}}{dz}, \tag{3}$$
$$z = x^{-1/2\nu} = \frac{(\xi_{ab0}^{\pm})^2 |t|^{-2\nu} H_c}{\Phi_0}.$$

$Q^{\pm}$ is a universal constant and $G^{\pm}(z)$ is a universal scaling function of its argument, with $G^{\pm}(z=0) = 1$. $\gamma = \xi_{ab}/\xi_c$ denotes the anisotropy, ξ_{ab} the zero-field in-plane correlation length and H_c the magnetic field applied along the c axis. In terms of the variable x the scaling form (3) is similar to Prange's [17] result for Gaussian fluctuations. Approaching T_c the in-plane correlation length diverges as

$$\xi_{ab} = \xi_{ab0}^{\pm}|t|^{-\nu}, \qquad t = T/T_c - 1, \qquad \pm = \mathrm{sgn}(t). \tag{4}$$

Supposing that 3D-xy fluctuations dominate, the critical exponents are given by [18]

$$\nu \simeq 0.671 \simeq 2/3, \qquad \alpha = 2\nu - 3 \simeq -0.013, \tag{5}$$

and there are the universal critical amplitude relations [1, 9–12, 18]

$$\frac{\xi_{ab0}^-}{\xi_{ab0}^+} = \frac{\xi_{c0}^-}{\xi_{c0}^+} \simeq 2.21, \qquad \frac{Q^-}{Q^+} \simeq 11.5, \qquad \frac{A^+}{A^-} = 1.07, \tag{6}$$

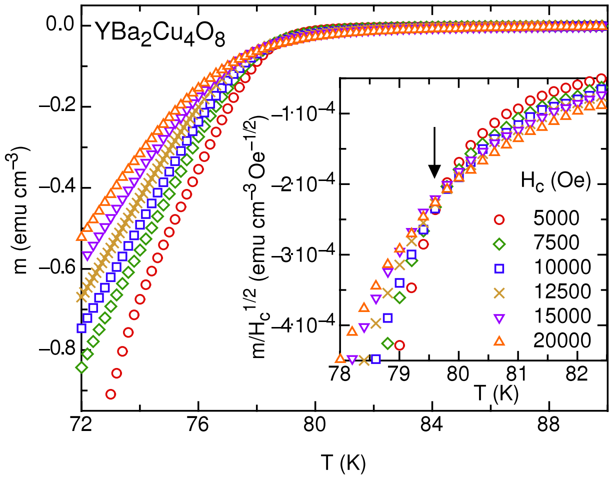

Figure 1. Reversible magnetization m versus T of a $YBa_2Cu_4O_8$ single crystal for magnetic fields H_c applied along the c axis. The inset shows $m/H_c^{1/2}$ versus T. The arrow indicates the crossing point which yields the estimate $T_c \simeq 79.6$ K.

and

$$A^-\xi_{a0}^-\xi_{b0}^-\xi_{c0}^- \simeq A^-(\xi_{ab0}^-)^2\xi_{c0}^- = \frac{A^-(\xi_{ab0}^-)^3}{\gamma}$$
$$= (R^-)^3, \qquad R^- \simeq 0.815. \tag{7}$$

$A^{\pm}$ is the critical amplitude of the specific heat singularity, defined as

$$c = \frac{C}{Vk_B} = \frac{A^{\pm}}{\alpha}|t|^{-\alpha} + B, \tag{8}$$

where B denotes the background. Furthermore, in the 3D-xy universality class T_c, ξ_{c0}^- and the critical amplitude of the in-plane magnetic field penetration depth λ_{ab0} are not independent, but related by the universal relation [1, 9–11, 18]

$$k_BT_c = \frac{\Phi_0^2}{16\pi^3}\frac{\xi_{c0}^-}{\lambda_{ab0}^2} = \frac{\Phi_0^2}{16\pi^3}\frac{\xi_{ab0}^-}{\gamma\lambda_{ab0}^2}. \tag{9}$$

Furthermore, the existence of the magnetization at T_c of the penetration depth below T_c and of the magnetic susceptibility above T_c imply the following asymptotic forms of the scaling function [1, 9–12]:

$$Q^{\pm}\frac{1}{\sqrt{z}}\frac{dG^{\pm}}{dz}\bigg|_{z\to\infty} = Q^{\pm}c_{\infty}^{\pm},$$
$$Q^-\frac{dG^-}{dz}\bigg|_{z\to 0} = Q^-c_0^-(\ln z + c_1), \tag{10}$$
$$Q^+\frac{1}{z}\frac{dG^+}{dz}\bigg|_{z\to 0} = Q^+c_0^+,$$

with the universal coefficients

$$Q^-c_0^- \simeq -0.7, \qquad Q^+c_0^+ \simeq 0.9, \qquad Q^{\pm}c_{\infty}^{\pm} \simeq 0.5, \tag{11}$$
$$c_1 \simeq 1.76. \tag{12}$$

We are now prepared to analyze the magnetization data. To estimate T_c we note that, according to equations (3), (10)

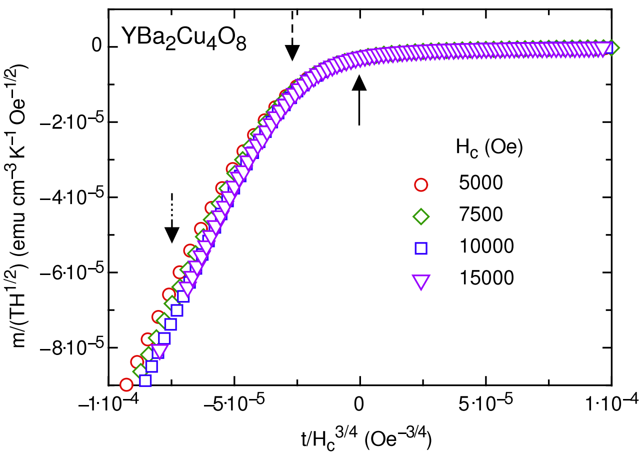

Figure 2. $m/(TH_c^{1/2})$ versus $t/H^{3/4}$ for a $YBa_2Cu_4O_8$ single crystal with $T_c = 79.6$ K. The full arrow marks the zero-field critical temperature T_c, the dashed arrow the 3D to 1D crossover and the dotted arrow the vortex melting line.

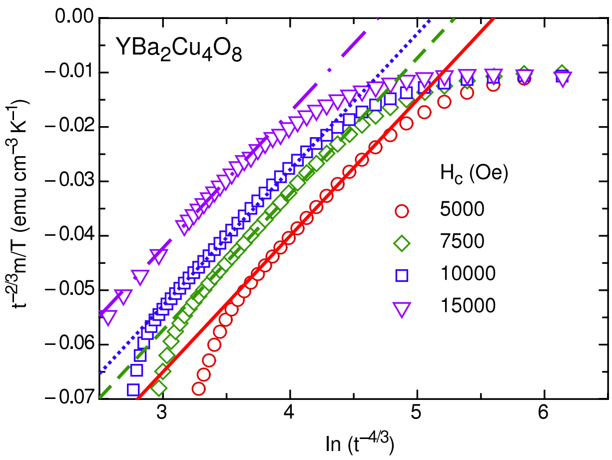

Figure 3. $|t|^{-2/3}m/T$ versus $\ln(|t|^{-4/3})$ for a $YBa_2Cu_4O_8$ single crystal according to equation (14). The lines are fits to the rescaled magnetization data. Here $\xi_{ab0}^2|t_p|^{-4/3} = \Phi_0/aH_c$.

and (12), the plot $m/H_c^{1/2}$ versus T should exhibit a crossing point at T_c because $m/TH_c^{1/2}$ tends to the value $m/T_cH_c^{1/2} = -0.5k_B\gamma\Phi_0^{-3/2}$. The inset in figure 1 reveals that there is a crossing point near $T_c \simeq 79.6$ K. Given this estimate, consistency with 3D-xy critical behavior then requires according to the scaling form (3) that the data plotted as $m/(TH_c^{1/2})$ versus $tH_c^{-1/2\nu} \simeq tH_c^{-3/4}$ should collapse near $tH_c^{-3/4} \to 0$ on a single curve. Evidence for this collapse emerges from figure 2 with $T_c \simeq 79.6$ K. Considering the limit $z \to 0$ below T_c the appropriate scaling form is

$$\frac{m}{T} = -\frac{Q^- c_0^- k_B}{\Phi_0 \xi_c^-}\left(\ln\left(\frac{H_c(\xi_{ab})^2}{\Phi_0}\right) + c_1\right), \tag{13}$$

according to equations (3), (10) and (12). Thus, given the magnetization data of a homogeneous system, attaining the limit $z = H_c(\xi_{ab0}^{\pm})^2|t|^{-2\nu}/\Phi_0 \ll 1$, the growth of ξ_{ab} and ξ_c is unlimited and estimates for ξ_{c0}^- and ξ_{ab0}^- can be deduced from

$$|t|^{-2/3}\frac{m}{T} = -\frac{Q^- c_0^- k_B}{\Phi_0 \xi_{c0}^-}\left(\ln\left(\frac{H_c(\xi_{ab0}^-)^2|t|^{-4/3}}{\Phi_0}\right) + c_1\right). \tag{14}$$

In figure 3 we depicted $|t|^{-2/3}m/T$ versus $\ln(|t|^{-4/3})$. From the straight lines we obtain

$$-\frac{Q^- c_0^- k_B}{\Phi_0 \xi_{c0}^-} \simeq 0.025, \tag{15}$$

and with that

$$\xi_{c0}^- \simeq 1.87 \text{ Å}. \tag{16}$$

Furthermore, from $\ln(H_c(\xi_{ab0}^-)^2/\Phi_0)$ versus $\ln(H_c)$ we deduce

$$\xi_{ab0}^- \simeq 15.6 \text{ Å}. \tag{17}$$

For the anisotropy we obtain then the estimate

$$\gamma = \frac{\xi_{ab0}^-}{\xi_{c0}^-} \simeq 8.34, \tag{18}$$

compared to $\gamma \simeq 13.4$, $\gamma \simeq 14.7$ [19] and $\gamma \simeq 12.3$ [20].

To explore the magnetic-field-induced 3D to 1D crossover further and to probe the vortex melting line directly we invoke Maxwell's relation

$$\left.\frac{\partial(C/T)}{\partial H_c}\right|_T = \left.\frac{\partial^2 M}{\partial T^2}\right|_{H_c}, \tag{19}$$

uncovering the vortex melting transition in terms of a singularity, while the magnetic-field-induced finite size effect leads to a dip. These features seem to differ drastically from the nearly smooth behavior of the magnetization. Together with the scaling form of the specific heat (equation (8)), extended to the presence of a magnetic field:

$$c = \frac{A^-}{\alpha}|t|^{-\alpha} f(x), \qquad x = \frac{t}{H_c^{1/2\nu}}, \tag{20}$$

we obtain the scaling form

$$\begin{aligned} TH_c^{1+\alpha/2\nu}\frac{\partial(c/T)}{\partial H_c} &= -\frac{k_B A^-}{2\alpha\nu}x^{1-\alpha}\frac{\partial f}{\partial x} \\ &= TH_c^{1+\alpha/2\nu}\frac{\partial^2 m}{\partial T^2}. \end{aligned} \tag{21}$$

In figure 4 we depicted $TH_c\,\mathrm{d}^2m/\mathrm{d}T^2$ versus x for various magnetic fields H_c. Apparently, the data collapses reasonably well on a single curve. There is a peak and a dip marked by an arrow and a vertical line, respectively. Their occurrence differ clearly from the traditional mean-field behavior where $\partial^2 m/\partial T^2 = 0$. The finite depth of the dip is controlled by the magnetic-field-induced finite size effect. It replaces the reputed singularity at the upper critical field obtained in the Gaussian approximation [17]. Note that both the peak and the dip are hardly visible in the magnetization shown in figure 2. There we marked the location of the peak, the dip and T_c in terms of dashed and solid arrows. The location of the dip determines the line

$$x_p = t_p H_c^{-3/4} \simeq -2.85 \times 10^{-5} \text{ Oe}^{-3/4}, \tag{22}$$

in the (H_c, T) plane where the 3D to 1D crossover occurs. Along this line, rewritten in the form $H_{pc}(T) = \Phi_0/(a(\xi_{ab0}^-)^2)(1 - T/T_c)^{4/3}$, the in-plane correlation length is

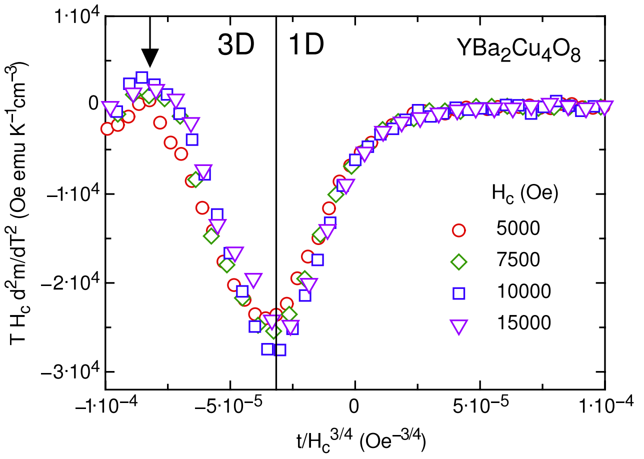

Figure 4. $TH_c\mathrm{d}^2m/\mathrm{d}T^2$ versus $x = t/H_c^{3/4}$ for a $YBa_2Cu_4O_8$ single crystal. The arrow marks the vortex melting line $x_m \simeq -8.35 \times 10^{-5}$ $(\mathrm{Oe}^{-3/4})$ and the vertical line $x_p \simeq -2.85 \times 10^{-5}$ $(\mathrm{Oe}^{-3/4})$ the 3D to 1D crossover line.

limited by L_{H_c} (equation (1)). In addition there is a peak at

$$x_m = t_m H_c^{-3/4} \simeq -8.35 \times 10^{-5}\ \mathrm{Oe}^{-3/4}, \qquad (23)$$

corresponding to the vortex melting transition. Rewritten, the vortex melting line follows in the form $H_{mc} \simeq 2.7 \times 10^5 \mathrm{Oe} \cdot (1 - T_m/T_c)^{4/3}$ which agrees very well with the previous estimate $H_{mc} \simeq 1.8 \times 10^5\ \mathrm{Oe} \cdot (1 - T_m/T_c)^{4/3}$ obtained by Katayama *et al* [21] as far as the temperature dependence is concerned. Accordingly, we obtain the universal ratios of the scaling variables of the reduced temperatures for the vortex melting line and the 3D to 1D crossover line as

$$\frac{z_m}{z_p} = \left(\frac{t_p(H_c)}{t_m(H_c)}\right)^{2\nu} \simeq 0.24, \qquad (24)$$
$$t_p(H_c)/t_m(H_c) \simeq 0.34.$$

These values agree well with the estimates $t_p(H_c)/t_m(H_c) \simeq 0.3$ for an $NdBa_2Cu_3O_{7-\delta}$ single crystal [22] and $t_p(H_c)/t_m(H_c) \simeq 0.35$ for a $YBa_2Cu_3O_{6.97}$ single crystal [23] derived from the respective references.

Finally, invoking the universal relation (9) we obtain with $T_c = 79.6$ K and $\xi_{c0}^- \simeq 1.87$ Å(equation (16)) for the critical amplitude of the in-plane magnetic field penetration depth the value $\lambda_{ab0} \simeq 1.37 \times 10^{-5}$ cm, in reasonable agreement with the estimate $\lambda_{ab0} \simeq 1.7 \times 10^{-5}$ cm obtained from magnetization data of polycrystalline $YBa_2Cu_4O_8$ samples [24].

3. Summary

We have shown that the analysis of reversible magnetization data of a $YBa_2Cu_4O_8$ single crystal provides considerable insight into the effect of thermal fluctuations and the magnetic-field-induced 3D to 1D crossover. In particular we demonstrated that the fluctuation-dominated regime is experimentally accessible and uncovers remarkable consistency with 3D-xy critical behavior. Furthermore there is, however, the magnetic-field-induced finite size effect. It implies that the correlation length transverse to the magnetic field H_i, applied along the i axis, cannot grow beyond the limiting magnetic length $L_{H_i} = (\Phi_0/(aH_i))^{1/2}$, related to the average distance between vortex lines. Invoking the scaling theory of critical phenomena, clear evidence for this finite size effect has been provided. In type II superconductors it comprises the 3D to 1D crossover line $H_{pi}(T) = (\Phi_0/(a\xi_{j0}^-\xi_{k0}^-))(1 - T/T_c)^{4/3}$ with $i \neq j \neq k$ and $\xi_{i0,j0,k0}^-$ denoting the critical amplitude of the correlation length below T_c. As a result, below T_c and above $H_{pi}(T)$ superconductivity is confined to cylinders with diameter L_{H_i} (1D). Accordingly, there is no continuous phase transition in the (H_c, T) plane along the H_{c2} lines as predicted by the mean-field treatment. In addition, we confirmed the universal relationship between the 3D to 1D crossover and vortex melting line. The universal relation (9) and Maxwell's relation (19) also imply that the effects of isotope exchange and pressure on T_c, in-plane magnetic field penetration depth, correlation lengths, specific heat and magnetization are not independent.

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